Exact Analysis of Rectangular Waveguides Inhomogeneously Filled with a Transversely Magnetized Semiconductor

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Abstract—**An exact solution for the complex propagation constant in semiconductor loaded waveguides is obtained by superimposing a finite number of plane waves. The analysis is carried out through the study of the parallel-plate waveguide. Numerical results have been obtained by means of a numerical program previously set up [18]. Reciprocal and nonreciprocal behavior of the electromagnetic (EM) structure, depending on the semiconductor parameters, geometrical parameters, and applied magnetic field, is illustrated. A good quantitative agreement between the theory and the experiments in [11] is shown.**

I. Introduction

In recent years several investigators have been concerned with electromagnetic (EM) wave propagation in rectangular waveguides, either partially or totally filled with a semiconductor, sometimes transversely magnetized.

Reciprocal propagation in rectangular waveguides totally filled with a transversely magnetized semiconductor has been studied by Engineer and Nag [1] in the limiting case of a small applied magnetic field: their conclusions have been confirmed by Rahman and Gunn [2]. A perturbation technique [3] has been used by Gabriel and Brodwin to obtain a first-order approximation for the waveguide totally filled with a transversely magnetized semiconductor [4]. Experimental studies have been performed by Barlow and Koike [5] and later by Toda [6]–[8], whose results have been interpreted theoretically by Hirota [9] by assuming the semiconductor conductivity very high in the direction of the applied magnetic field. Hirota and Suzuki [10] have developed a variational analysis, complete with experimental data, of a rectangular waveguide loaded with a very thin slab of transversely magnetized semiconductor. Nonreciprocal propagation in the partially filled waveguide has been demonstrated by Arnold and Rosenbaum [11] through an expansion of the field in terms of TE_{10} and TM_{11} empty waveguide modes and the use of Schelkunoff's telegraphist's equations [12]. However, Sheikh and Gunn [13] on one hand, and Vander Vost and Govaerts [14] on the other, have pointed out the limits of the approximation techniques used for this kind of structure. An exact analysis for solving the rectangular waveguide loaded with any number of anisotropic slabs has been developed by Gardiol [15]. Nevertheless, the particular case of the guide loaded with one slab of a transversely magnetized semiconductor can be solved by means of a simpler technique, i.e., by superimposing a finite number of plane waves. This technique is substantially equivalent to that of Gardiol, and it is similar to that developed by Barzilai and Gerosa [16] for the analogous structure containing magnetized ferrite.

In Section II the characteristic equation for the propagation constant of the modes is worked out through an analysis of the parallel-plate waveguide. In Section III the numerical results obtained by employing the tensor permittivity given by Engineer and Nag [1] are presented and discussed together with a comparison with the experiments of Arnold and Rosenbaum [11].

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II. The Parallel-Plate Waveguide and the Rectangular Waveguide

For a semiconductor in the presence of a steady magnetic field $\boldsymbol{B}_0$ oriented along the z axis, the complex permittivity tensor is

$$\bar{\varepsilon} = \varepsilon \begin{bmatrix} \varepsilon_1 & -\varepsilon_3 & 0 \\ \varepsilon_1 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_2 \end{bmatrix} \tag{1}$$

for a time-harmonic dependence of the fields. ε is the semiconductor permittivity in the absence of the applied magnetic field. Generally, the elements of $\bar{\varepsilon}$ are a function of the signal frequency and, with the exception of ε_2, also of $\boldsymbol{B}_0$. It is worth noting that the permittivity tensor presents a symmetry with respect to the z axis, so that waves traveling in the positive and negative z directions have the same phase velocities [15].

Let us consider the structures in Fig. 1(a) and (b). The modal solutions of the parallel-plate waveguide enable one, as we shall see, to construct the modes of the rectangular waveguide. Assuming the spatial dependence of the fields in the semiconductor ($y > 0$) to be $\exp(j\omega\sqrt{\mu\varepsilon}\,\boldsymbol{n}\cdot\boldsymbol{r})$, the condition that homogeneous Maxwell equations possess nontrivial solutions leads to the dispersion relation

$$n^4 + (\varepsilon_2/\varepsilon_1 - 1)n_z^2 n^2 - (\varepsilon_2 + \varepsilon_{\text{eff}})n^2 - (\varepsilon_2 - \varepsilon_{\text{eff}})n_z^2 + \varepsilon_2\varepsilon_{\text{eff}} = 0 \tag{2}$$

where

$$n^2 = \boldsymbol{n}\cdot\boldsymbol{n} = n_x^2 + n_y^2 + n_z^2$$

$$\varepsilon_{\text{eff}} = (\varepsilon_1^2 + \varepsilon_3^2)/\varepsilon_1.$$

The expressions of the EM field, which can be deduced from Maxwell's equations, are, not considering an arbitrary

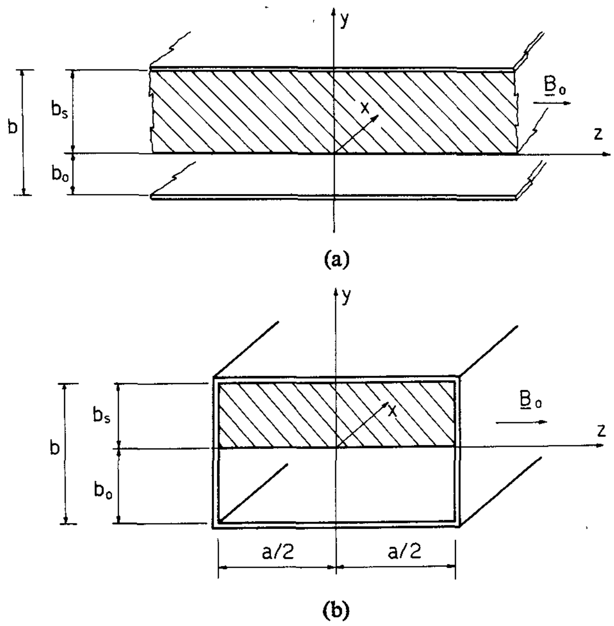


Fig. 1. Geometry of the parallel-plate and rectangular waveguides.

factor,

$$E = \varepsilon_1(\varepsilon_{\text{eff}} - n^2)z_0 - n_z(\varepsilon_1 - n^2)n + \varepsilon_3 n_z z_0 \times n$$

$$\eta H = -\varepsilon_1(\varepsilon_{\text{eff}} - n^2)n \times z_0 + \varepsilon_3 n_z(n_z n - n^2 z_0) \qquad (3)^1$$

where $\eta = \sqrt{\mu/\varepsilon}$.

In a vacuum ($y < 0$), the normalized wave vector will be

$$k = n_x x_0 + k_y y_0 + n_z z_0.$$

From Maxwell's equations it follows that a generic EM field in the vacuum can be expressed as a combination of the two fields

$$e_1 = n_z(n_x k - k^2 x_0)$$

$$\eta h_1 = \frac{\varepsilon_0}{\varepsilon} n_z k \times x_0 \qquad (4a)$$

$$e_2 = k \times x_0$$

$$\eta h_2 = \frac{\mu}{\mu_0}(k^2 x_0 - n_x k) \qquad (4b)$$

with

$$k = k \cdot k = \mu_0\varepsilon_0/\mu\varepsilon. \qquad (5)$$

In (4a) and (4b) spatial dependence $\exp(j\omega\sqrt{\mu\varepsilon}k \cdot r)$ is understood. The field (4a) vanishes for $n_z = 0$, as it must when the parallel-plate guide is closed by two perfectly conducting planes in such a way as to obtain the rectangular waveguide.

For each couple n_x, n_z, (2) gives four values of n_y:

$$\pm n_{y1} \qquad \pm n_{y2}$$

[1] These expressions fail if either $n_z = 0$ or $n_y = \pm jn_x$; in these cases the right-hand sides of (3) vanish. For the sake of brevity we omit the expressions to be adopted in these cases instead of (3).

while, in the vacuum, (5) has two roots for k_y:

$$\pm k_y.$$

Therefore, in accordance with [17], we may write

1) EM field in the semiconductor ($y > 0$):

$$\begin{Bmatrix} E \\ H \end{Bmatrix} = \left[\sum_{i=1}^{2} A_i^+ \begin{Bmatrix} E_i^+ \\ H_i^+ \end{Bmatrix} \exp(j\omega\sqrt{\mu\varepsilon}\, n_{y_i} y) + A_i^- \begin{Bmatrix} E_i^- \\ H_i^- \end{Bmatrix} \exp(-j\omega\sqrt{\mu\varepsilon}\, n_{y_i} y)\right] \cdot \exp[j\omega\sqrt{\mu\varepsilon}(n_x x + n_z z)] \qquad (6)$$

where $E_i^\pm, H_i^\pm$ are obtained from (3) by putting $n_y = \pm n_{yi}$.

2) EM field in the vacuum ($y < 0$):

$$\begin{Bmatrix} E \\ H \end{Bmatrix} = \left[\sum_{i=1}^{2} B_i^+ \begin{Bmatrix} e_i^+ \\ h_i^+ \end{Bmatrix} \exp(j\omega\sqrt{\mu\varepsilon}\, k_y y) + B_i^- \begin{Bmatrix} e_i^- \\ h_i^- \end{Bmatrix} \exp(-j\omega\sqrt{\mu\varepsilon}\, k_y y)\right] \cdot \exp[j\omega\sqrt{\mu\varepsilon}(n_x x + n_z z)] \qquad (7)$$

where $e_i^\pm, h_i^\pm$ are obtained from (4a) and (4b) by putting $k_y = \pm k_y$. The boundary conditions at $y = -b_0$ and $y = b_s$ and the interface condition at $y = 0$ lead to a homogeneous system of eight algebraic equations in the eight unknowns $A_i^\pm, B_i^\pm$. The condition for the existence of a nontrivial solution leads to the characteristic equation of the structure.

Let us now consider the rectangular waveguide. It can be seen from (3) and (4) that the boundary conditions at $z = \pm a/2$ are satisfied by the superimposition of two parallel-plate guide modes having the same amplitude and opposite phase velocities in the z direction and with

$$n_z = \frac{m\pi}{a}\frac{1}{\omega\sqrt{\mu\varepsilon}}, \qquad m = 1,2,3,\cdots. \qquad (8)$$

The case $m = 0$ (thus $n_z = 0$) is of no interest in the present study. In this case, in fact, the structure only supports TE modes which are independent of the applied magnetic field: the semiconductor behaves as an isotropic medium with complex permittivity ε_2. The values of the longitudinal propagation constant may therefore be obtained from the parallel-plate case with condition (8). The characteristic equation of the structure may be written, after some manipulations,

$$\begin{vmatrix} 0 & 0 & 0 & 0 & \beta^+ & \beta^- & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & k_y\beta^+ & -k_y\beta^- \\ E_{x1}^+ & E_{x1}^- & E_{x2}^+ & E_{x2}^- & n_z(k_y^2+n_z^2) & n_z(k_y^2+n_z^2) & 0 & 0 \\ E_{z1}^+ & E_{z1}^- & E_{z2}^+ & E_{z2}^- & -n_z^2 n_x & -n_z^2 n_x & k_y & -k_y \\ E_{x1}^+\alpha_1^+ & E_{x1}^-\alpha_1^- & E_{x2}^+\alpha_2^+ & E_{x2}^-\alpha_2^- & 0 & 0 & 0 & 0 \\ E_{z1}^+\alpha_1^+ & E_{z1}^-\alpha_1^- & E_{z2}^+\alpha_2^+ & E_{z2}^-\alpha_2^- & 0 & 0 & 0 & 0 \\ \eta H_{x1}^+ & \eta H_{x1}^- & \eta H_{x2}^+ & \eta H_{x2}^- & 0 & 0 & -\frac{\mu}{\mu_0}(k_y^2+n_z^2) & \frac{-\mu}{\mu_0}(k_y^2+n_z^2) \\ \eta H_{z1}^+ & \eta H_{z1}^- & \eta H_{z2}^+ & \eta H_{z2}^- & \frac{\varepsilon_0}{\varepsilon}k_y n_z & \frac{-\varepsilon_0}{\varepsilon}k_y n_z & \frac{\mu}{\mu_0}n_z n_x & \frac{\mu}{\mu_0}n_x n_z \end{vmatrix} = 0 \quad (9)$$

where

$$\alpha_i^{\pm} = \exp(\pm j\omega\sqrt{\mu\varepsilon}\, n_{yi} b_s), \qquad \beta^{\pm} = \exp(\mp j\omega\sqrt{\mu\varepsilon}\, k_y b_0).$$

The characteristic equation of the completely filled guide is obtained by taking the determinant inside the dashed line as zero. In this case the propagation is reciprocal: it is not difficult to see that this determinant does not change value by altering the sign of n_x.

For each value of m, (2) and (5) allow one to express n_{yi} and k_y as functions of n_x; therefore through (3), (9) is reduced to an equation in the only unknown n_x:

$$f(n_x) = 0. \qquad (10)$$

This function is a two-valued analytical one since its argument is determined apart from any consideration of π. This fact occurs since the functions $n_{yi}(n_x)$ and $k_y(n_x)$, implicitly defined by (2) and (5), are two valued. In searching for the solutions of (10), suitable precautions should therefore be adopted because of the existence of the branch points. These critical points correspond, in Gardiol's analysis, to the values of γ for which the matrix $[A]$ has a null eigenvalue of multiplicity two.

III. Results

When microwave frequency is much lower than carrier collision frequency, the form of the complex permittivity tensor given by Engineer and Nag [1] may be adopted:

$$\varepsilon_1 = 1 - j\frac{\sigma}{\omega\varepsilon[1 + (\sigma R B_0)^2]}$$

$$\varepsilon_2 = 1 - j\sigma/\omega\varepsilon$$

$$\varepsilon_3 = j\frac{\sigma^2 R B_0}{\omega\varepsilon[1 + (\sigma R B_0)^2]} \qquad (11)$$

where R is the Hall constant of the semiconductor and σ is the dc conductivity.

For this case the normalized propagation constant n_x of the modes of order $m = 1$ has been calculated. The X-band waveguide dimensions have been assumed ($a = 22.86$ mm, $b = b_0 + b_s = 10.16$ mm).

The solutions of (10) have been sought by means of a numerical program [18] based on the well-known formula valid for an analytical function:

$$\sum_i z_i^N = \frac{1}{2\pi j}\oint_{+\partial D} z^N \frac{f'(z)}{f(z)}\, dz$$

z_i being the zeros of $f(z)$ inside the domain D of the complex plane z. This method presents the advantage of allowing the determination of the zeros without having to evaluate the function in their proximity. This results in a very high precision which, in some cases, appears to be necessary.

Fig. 2(a) and (b) shows the distribution of the zeros of $f(n_x)$ for two values of the filling ratio b_s/b. The zeros are approximately symmetrical with respect to the origin: this means that the wave propagation is approximately reciprocal for the parameter values indicated in Fig. 2. Most of the zeros are close to the imaginary axis, which shows that most of the modes are strongly attenuated. This is not surprising because, in the empty waveguide, only the fundamental can propagate at the frequency of 10 GHz. Nevertheless, as the filling ratio increases, two pairs of zeros approach the real axis. This behavior is clarified in Fig. 3 where the normalized phase constant (real part of n_x) and attenuation constant (imaginary part) of the first six modes (three positively and three negatively traveling) are reported versus b_s/b. It is worth noting that the attenuation does not vary monotonically with the filling ratio. This behavior, which also occurs in waveguides partially filled with a lossy dielectric, has already been noticed by [11] and interpreted by Gardiol and Parriaux [19] as due to a large concentration of the electric field within the dissipative medium. The following effect is also due to the variable concentration of the electric field in the slab. The modes a and d, corresponding to the fundamental for $b_s/b = 0$, present a lower attenuation than the others as long as $b_s/b < \sim 0.45$; for values of the filling ratio between ~ 0.45 and ~ 0.90 it is the modes c and f which have the lowest attenuation. Let us finally note that the values of the

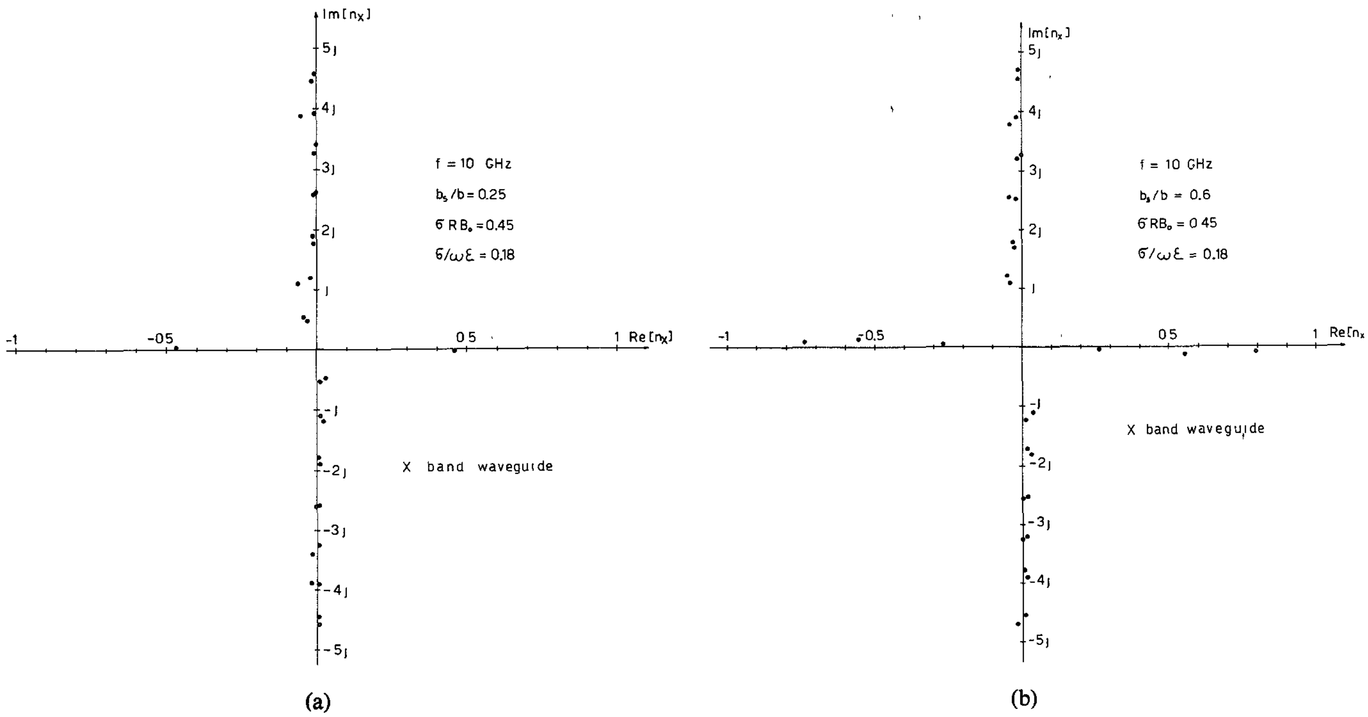


Fig. 2. Distribution of the zeros of $f(n_x)$ in the complex plane n_x for $b_s/b = 0.25$ and $b_s/b = 0.60$.

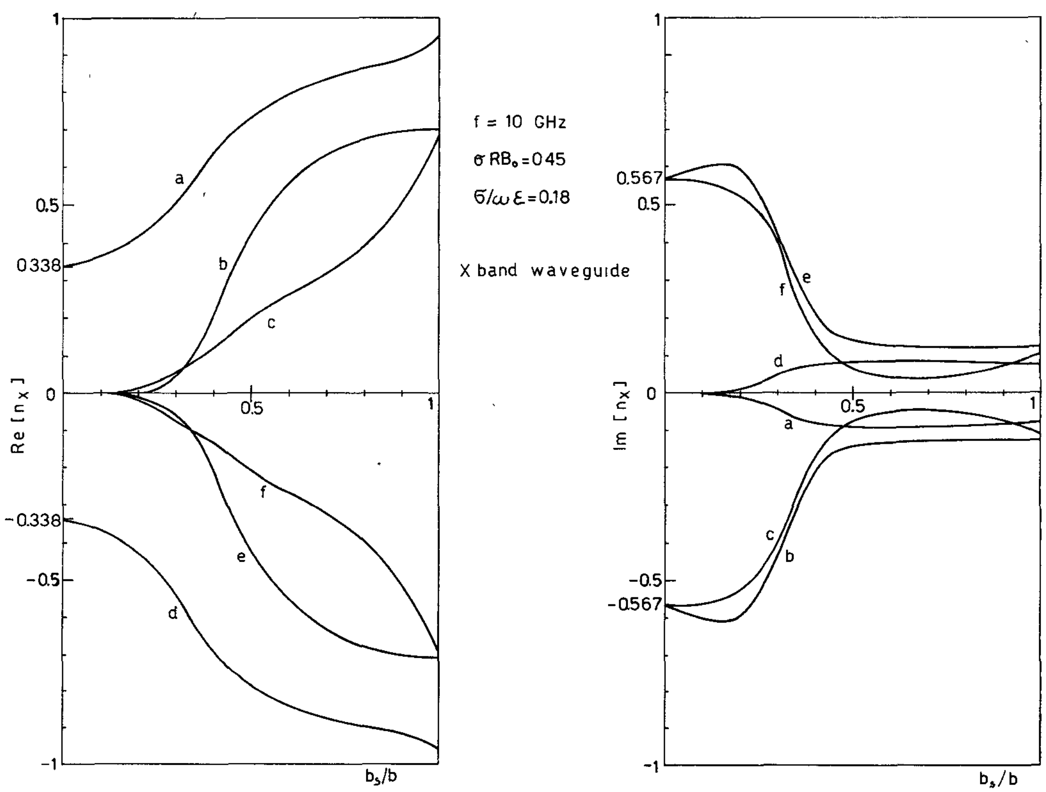


Fig. 3. Normalized phase constants and attenuation constants of the first six modes versus the filling ratio.

propagation constant of the modes are exactly opposite only for $b_s/b = 0$ and for $b_s/b = 1$, i.e., in the cases of the empty and totally filled waveguide. When the thickness b_s of the slab tends to zero, the six modes tend to the TE_{10}, TE_{11}, TM_{11} modes of the empty waveguide, for which

$$n_{xTE_{10}} = \pm 0.338 \qquad n_{xTE_{11}} = n_{xTM_{11}} = \pm j0.567$$

at the frequency of 10 GHz.

In Fig. 4 the first six modes (i.e., those with the lowest attenuation) are still considered, but for different values

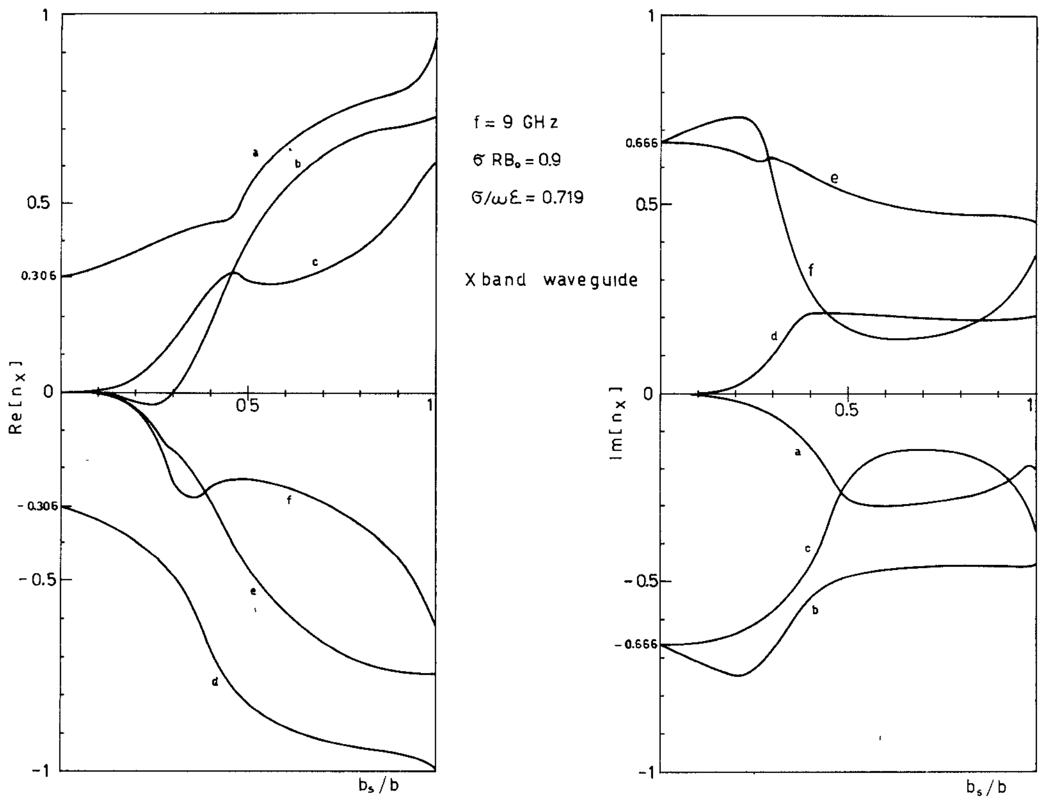


Fig. 4. The same as Fig. 3 except for $f = 9$ GHz and $\sigma RB_0 = 0.9$.

of the signal frequency and, particularly, of σRB_0. In this case the propagation appears to be completely nonreciprocal, in accordance with the fact that the off-diagonal elements of the tensor permittivity are greater than in the preceding case. In fact, for a fixed $\sigma/\omega\varepsilon$, (11) shows that ε_3 reaches its maximum value for $\sigma RB_0 = 1$. It is interesting to note that the mode b propagates in the positive x direction for $b_s/b < 0.3$, while, for greater values of the filling ratio, it propagates with a negative phase velocity. On the contrary, this mode is always attenuated in the negative x direction. The calculation of the group velocity, however, shows that it remains negative for all values of b_s/b: for $b_s/b < 0.3$ we are therefore dealing with a backward wave [20].

In Fig. 5 the real and imaginary parts of n_x are reported versus the frequency in the range 5–35 GHz and for $b_s/b = 0.5$. One can note that at lower frequencies, i.e., for greater $\sigma/\omega\varepsilon$, the nonreciprocal behavior of the structure becomes relevant. On the contrary, as the frequency increases, the semiconductor tends to behave as an isotropic medium and the propagation tends to be reciprocal: this is consistent with the fact that, for $\omega \to \infty$, (11) becomes

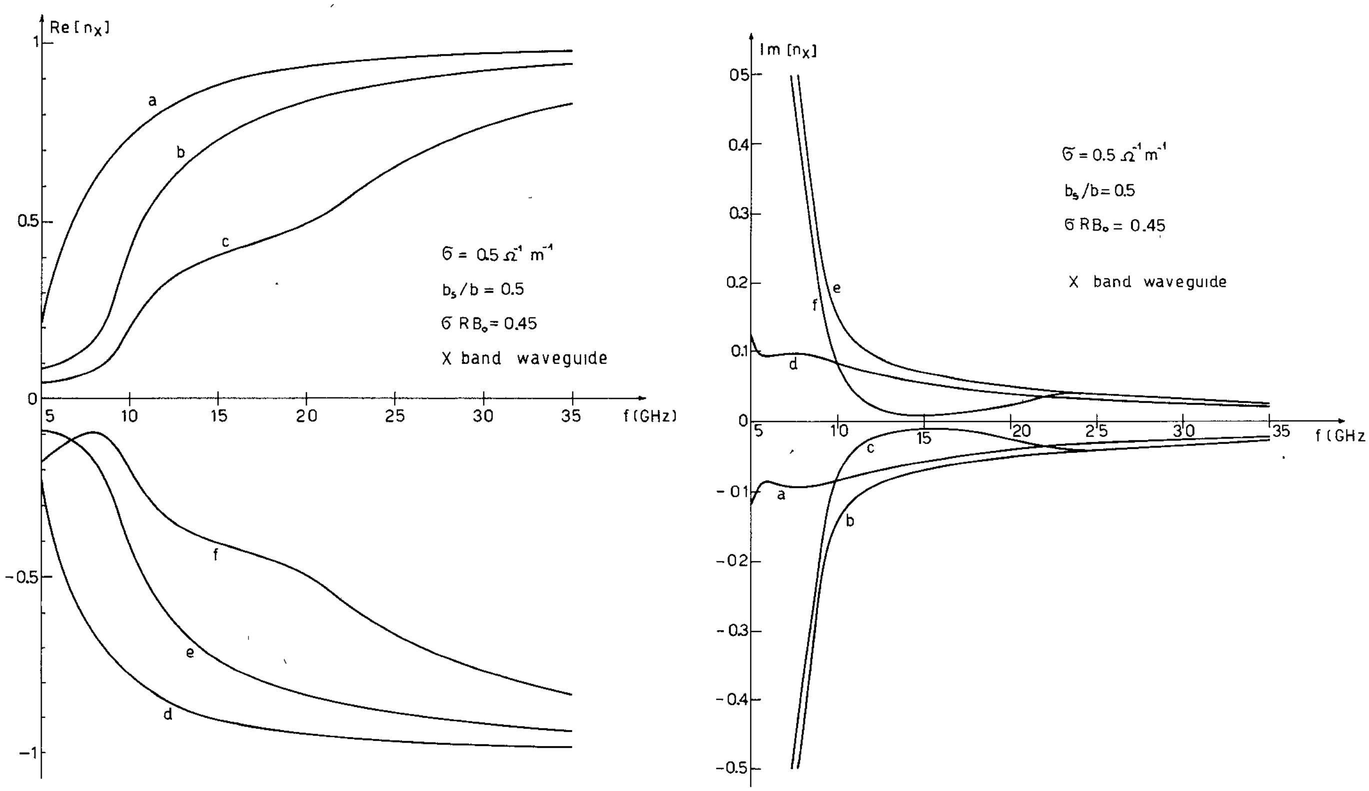
$$\varepsilon_1 = \varepsilon_2 = 1$$
$$\varepsilon_3 = 0.$$

The most remarkable effect in Fig. 5 is that the normalized phase constant of the mode f undergoes a change of the slope at about 8 GHz. The calculation of the group velocity shows that it is actually directed like the phase velocity, i.e., in the positive x direction, for $f > \sim 7.5$ GHz; instead, below this frequency it is opposite to the phase velocity. In this case also, below ~ 7.5 GHz, we are therefore dealing with a backward wave.

Fig. 6 shows n_x of the first six modes as a function of RB_0. When $RB_0 = 0$ one obtains the modes of the waveguide loaded with a lossy dielectric with complex permittivity ε_2. The action of the magnetic field results in a reduced attenuation of the modes a and d. These modes, which correspond to the fundamental in the empty waveguide (see Fig. 4), present a greater attenuation than the modes c and f, respectively, for $RB_0 < \sim 0.6$. On the contrary, their attenuation decreases with increasing magnetic field. In fact, it is easily seen that the conduction current, by increasing B_0, tends to be oriented in the z direction: consequently, the conduction losses $\frac{1}{2}\boldsymbol{E} \cdot \boldsymbol{J}^*$ decrease for the modes which have a smaller z component of the electric field and which, therefore, are particularly influenced by the value of the steady magnetic field. It is derived that a TE_{10} mode of the empty waveguide will preferably excite the modes a or d, depending on the direction of propagation and/or on the sign of B_0. This observation will be useful in the following.

For small values of RB_0 one can note from Fig. 6 that the attenuation increases with increasing magnetic field for the negatively traveling mode a, and decreases for the mode d which propagates in the opposite direction. This is consistent with the experimental results presented in [5] and [11].

Fig. 7 shows a comparison between the present theory (continuous line) and the experiments of Arnold and



(a) (b)

Fig. 5. Normalized phase constants and attenuation constants of the first six modes versus the frequency (gigahertz).

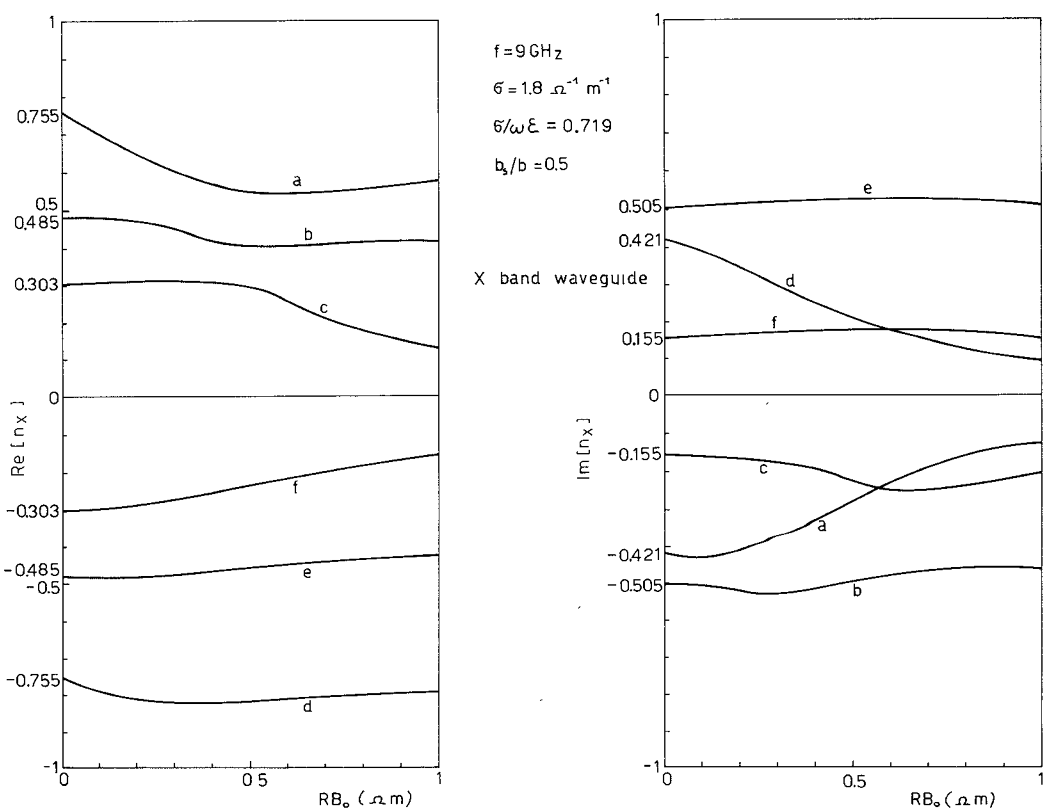


Fig. 6. Normalized phase constants and attenuation constants of the first six modes versus RB_0 for $b_s/b = 0.5$.

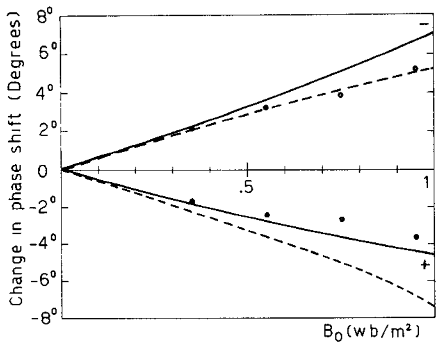


Fig. 7. Comparison between the present theory (continuous line) and the theory (dashed line) and the experiment of Arnold and Rosenbaum for change in phase shift as a function of magnetic field.

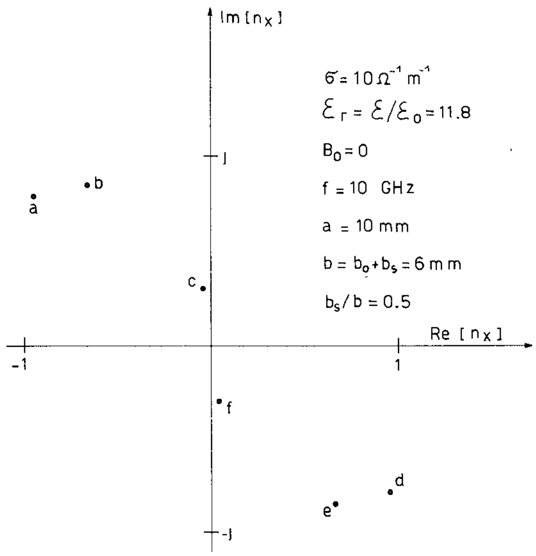


Fig. 8. Distribution of the zeros of $f(n_x)$ in the conditions of the experiment of Arnold and Rosenbaum for $B_0 = 0$.

Rosenbaum [11]. They have measured the phase shift due to the applied magnetic field in a waveguide section of length 6.5 mm partially loaded with n-type silicon. An EM field coming from a waveguide totally filled with a lossless dielectric was launched into the semiconductor loaded waveguide. Particular precautions have been adopted to avoid multimode propagation. The dashed line represents their theoretical results. An exact solution of the problem of the incidence of an EM field on the semiconductor loaded waveguide would require the matching of the incident field with a superposition of the modes of the loaded waveguide. This observation may suggest a qualitative explanation of the imperfect agreement between our theoretical results and the experimental ones. The former, in fact, have been obtained under the assumption of the propagation of one single mode. Let us consider Fig. 8, where the values of n_x of the first modes are reported in the case $B_0 = 0$. As one could see, the modes a and d are the only ones which are appreciably influenced by the magnetic field and whose attenuation decreases for high values of B_0. Consequently, as far as we have seen with regard to Fig. 6, the modes a and d were preferably excited in the experiments. We have therefore obtained the theoretical results in Fig. 7 under the assumption of the propagation of the modes a or d; nevertheless, in the experiment the other modes could have been slightly excited. This explanation is consistent with the fact that the disagreement is relatively the same for both positively and negatively traveling waves. On the contrary, the theoretical results of Arnold and Rosenbaum agree very well with the measurements in the case of the negatively traveling wave, but they are erratic for the positively traveling one. This is clearly due to the approximation of their theory.

IV. Conclusions

The characteristic equation of the rectangular waveguide partially loaded with a transversely magnetized semiconductor has been derived and solved by means of a computer program previously set up [18]. This has been done by assuming the expression of the semiconductor permittivity given by Engineer and Nag [1], but other models could be adopted (e.g., the Drude–Zener model [21]).

The nonreciprocal properties of the structure have been illustrated in various cases, namely by varying the filling ratio, the frequency, and the applied magnetic field. Nonreciprocal propagation has been particularly shown for the values of $\sigma R B_0$ close to unity. For high frequencies and/or for low conductivities the propagation tends to be reciprocal. The behavior of the attenuation for small values of the magnetic field is consistent with the experimental results presented in [5] and [11]. A good agreement of the theory and the experiments in [11] has been shown through a quantitative comparison between them.

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Short Papers

On a Direct Use of Edge Condition in Modal Analysis

C. VASSALLO

Abstract—**The edge condition allows us to know the asymptotic decrease of modal amplitudes in some discontinuity problems in waveguides. One may take a direct account of this information in modal analysis and gain a significant improvement of the calculation when the field singularity at edge is important. The accuracy and the validity of this method are studied in two cases: the diaphragm and the junction between an empty waveguide and a partially dielectric-filled waveguide.**

Introduction

The modal analysis is appropriate for all the waveguide discontinuities contained in a single cross-section plane, i.e., discontinuities like irises or abrupt transitions from one kind of guide to another one [1]. Its formulation is very easy, and modern computers can cope with the high-rank linear systems which may result from its application. However, these systems are only the truncated approximations of the theoretical systems of infinite rank in a rigorous formulation of the method, and some difficulties, such as the relative convergence effect, may lead to false results [2], [3] or a too slow rate of convergence may lead to inaccurate results. In this work we present a method based upon the edge effect theory, which may improve the convergence. We shall present our method in Section I, then we shall study its application to different kinds of discontinuities in order to know its range of interest.

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I. Modal Analysis and Edge Effect

Let us consider an abrupt transition between a left waveguide the kth normal mode of which has transverse components (e_k', h_k'), and a right waveguide the pth normal mode of which has transverse components (e_p'', h_p''). The equations which describe the scattering of the nth left normal mode on the transition have the following form:

$$\sum_k (\delta_{kn} + R_k) e_k'(x,y) = \sum_p T_p e_p''(x,y) \qquad (1)$$

$$\sum_k (\delta_{kn} - R_k) h_k'(x,y) = \sum_p T_p h''(x,y) \qquad (2)$$

where the unknown coefficients are (R_k) and (T_p) $(k,p = 1,2,\cdots)$. By taking the cross product of the two sides of these equations with the functions of any set complete on the cross section, one obtains an equivalent infinite algebraic linear system. For instance, with the set $\{e_p''\}$ one may transform (1) into

$$T_p = \sum_{k=1}^{\infty} (\delta_{kn} + R_k) V_{pk}, \qquad (p = 1,2,\cdots,\infty) \qquad (3)$$

where the V_{pk} are defined by integrals on the mode components. Equation (2) is transformed in a similar way.

The practical resolution consists of retaining a finite number of unknown modal coefficients. For instance, system (3) is replaced by

$$T_p = \sum_{k=1}^{N} (\delta_{kn} + R_k) V_{pk}, \qquad (p = 1,2,\cdots,P) \qquad (4)$$

and the integers N and P are chosen in order to have as many equations as unknown coefficients. Then, one has an ordinary linear system.